Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Differential Geometry

Subject Code: 5SC02DIG1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date :16/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Write Cartesian representation of the curve$\bar{\gamma}(t) = (t, t)$. **(01)**

b. Give an example of a surface which cannot be covered by single surface patch. **(01)**

c. Write surface patch of plane. **(01)**

d. State Four vertex theorem. **(01)**

e. State Isoperimetric inequality. **(01)**

f. Write Frenet-Serret equation. **(02)**

Q-2 **Attempt all questions** **(14)**

a. Compute curvature and torsion of the curve $\bar{r}(t) = (e^t \cos t, e^t \sin t, e^t)$. **(05)**

b. Let $\bar{\gamma}$ be a regular curve in R^3 with nowhere vanishing curvature. Then $\bar{\gamma}$ is planar if and only if the torsion of $\bar{\gamma}$ is identically zero. **(05)**

c. If $\bar{\gamma}$ is a regular curve in R^3, then show that its curvature K is given by the formula $K = \frac{\|\ddot{\bar{\gamma}} \times \dot{\bar{\gamma}}\|}{\|\dot{\bar{\gamma}}\|^3}$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Wirtinger's inequality. **(05)**

b. let a and b be positive reals by applying isoperimetric inequality to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $\int_0^{2\pi} \sqrt{(a^2 \sin^2 t + b^2 \cos^2 t)}\, dt \geq 2\pi\sqrt{ab}$. **(05)**

c. Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a convex curve. **(04)**

Q-3 **Attempt all questions** **(14)**

a. Compute unit normal and first fundamental form of **(05)**

$$\sigma(u, v) = (u \cos v,\ u \sin v, v).$$

	b.	If $f: S_1 \to S_2$ and $g: S_2 \to S_3$ are smooth maps then show that $D_p(g \circ f)(p) = D_{f(p)} g\, D_p f \,, \forall p \in S_1$	**(05)**
	c.	Let $\sigma: U \to \boldsymbol{R}^3$ be a patch of a surface S containing a point $p = \sigma(u_0, v_0)$ for some $(u_0, v_0) \in U$ then show that the tangent space to S at p is a vector subspace of $\boldsymbol{R}^3$ spanned by $\sigma_u(u_0, v_0)$ and $\sigma_v(u_0, v_0)$.	**(04)**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	Let $\bar{\gamma}: (a, b) \to R^3$ be a regular curve with nowhere vanishing curvature. Then prove that the torsion τ of $\bar{\gamma}$ is $\frac{(\dot{\bar{\gamma}} \times \ddot{\bar{\gamma}}) \cdot \dddot{\bar{\gamma}}}{\left\| \dot{\bar{\gamma}} \times \ddot{\bar{\gamma}} \right\|^2}$.	**(06)**
	b.	Calculate the surface area of sphere of radius r.	**(05)**
	c.	Compute the signed unit normal of curve $\bar{\gamma}(t) = (e^{kt} \cos t \,, e^{kt} \sin t)$.	**(03)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: Mean curvature and Gaussian curvature.	**(02)**
	b.	State Bonnet's theorem.	**(02)**
	c.	Define Gauss map.	**(01)**
	d.	Define : Geodesic	**(01)**
	e.	Define: Umbilical point.	**(01)**
Q-5		**Attempt all questions**	**(14)**
	a.	Let $\sigma: U \to \boldsymbol{R}^3$ be a surface patch of an oriented surface S and let $p = \sigma(u, v)$ be point on S then prove that the matrix of W_p with respect to the basis $\{\sigma_u, \sigma_u\}$ of $T_p S$ is $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$.	**(05)**
	b.	State and prove Euler's theorem.	**(05)**
	c.	Compute Gaussian curvature and mean curvature of $\sigma(u, v) = (\cos u \,, \sin u \,, v)$	**(04)**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Compute principal curvature and principal vectors of surface given by $\sigma(u, v) = (u, v, uv)$ in the direction(0,0,0).	**(06)**
	b.	State and prove Meusnier's theorem.	**(05)**
	c.	Let σ be a surface patch of an oriented surface with the unit normal $\bar{N}$ then prove that $\bar{N}_u\, \sigma_u = -L, \bar{N}_u\, \sigma_v = -M$ and $\bar{N}_v \sigma_v = -N$.	**(03)**
Q-6		**Attempt all questions**	**(14)**
	a.	Let $\phi: U \to V$ be a diffeomorphism between open subsets of R^2. Let $\phi(u, v) = (f(u, v), g(u, v))$ where f and g are smooth functions. Prove that ϕ is conformal iff $(f_u = g_v \;\&\; f_v = -g_u)$ or $(f_u = -g_v \;\&\; f_v = g_u)$.	**(06)**
	b.	Find the image of Gauss map for $\sigma(u, v) = (u, v, u^2 + v^2), \ \forall \ u, v \in R$.	**(04)**
	c.	State and prove Gauss Theorem Egregium.	**(04)**

OR

Q-6 **Attempt all Questions** **(14)**

a. Compute the Christoffel's symbol of second kind for **(05)**

$$\sigma(u,v) = (u\cos v\,, u\sin v\,, u).$$

b. State Gauss – Bonnet theorem. Prove that the sum of interior angles of a regular $n-$ gon on a plane is $(n-2)\pi$ **(05)**

c. let σ be a regular surface patch then prove that **(04)**

$$\Gamma_{11}^{1} = \frac{GE_u - 2FF_u + FE_v}{2(EG-F^2)}, \text{ and } \Gamma_{11}^{2} = \frac{2EF_u - EE_v - FE_u}{2(EG-F^2)},$$